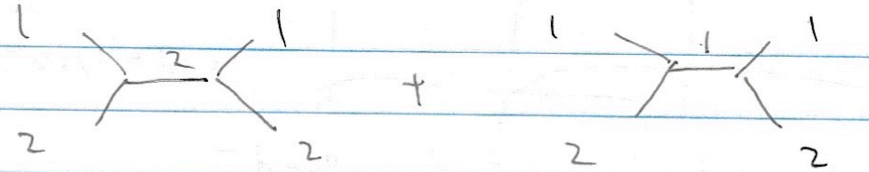
Schwartz 7.6 (in progress)

$$\mathcal{L}_{int} = \frac{\lambda}{2} \phi_1 (\partial_\mu \phi_2)(\partial_\mu \phi_2) + \frac{g}{2} \phi_1^2 \phi_2$$

There are two types of diagrams at tree level for $\phi_1 \phi_2 \to \phi_1 \phi_2$

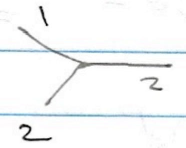


$$\phi_2 = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[a_k e^{ikx} + a_k^\dagger e^{-ikx} \right]$$

$$\partial_\mu \phi_2 = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[ik_\mu a_k e^{ikx} - ik_\mu a_k^\dagger e^{-ikx} \right]$$

$\Rightarrow$ annihilated ϕ_2 particle picks up ik_μ, created ϕ_2 particle picks up $-ik_\mu$,

For our concern, we only need to consider the

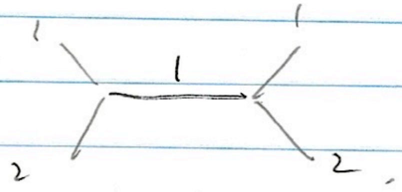


vertex, where one ϕ_2 is annihilated, and one ϕ_2 is created, so we get an overall factor

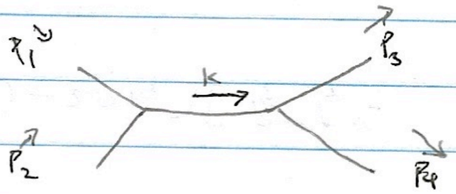
$$[ik_\mu(\text{incoming})][-ik_\mu(\text{outgoing})]$$

$$= k_\mu(\text{incoming})\, k^\mu(\text{outgoing})$$

Now for the diagram



In p-space:



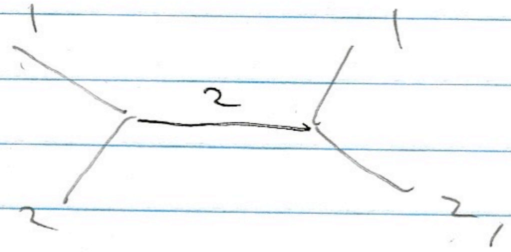
$$i\mathcal{M}_2 = ig \frac{i}{k^2 + i\varepsilon} ig \; \delta(p_1 + p_2 - k) \; \delta(k - p_3 - p_4)$$

$$\mathcal{M}_2 = -g^2 \frac{1}{(p_1+p_2)^2 + i\varepsilon} \times \delta^4(\Sigma p)$$

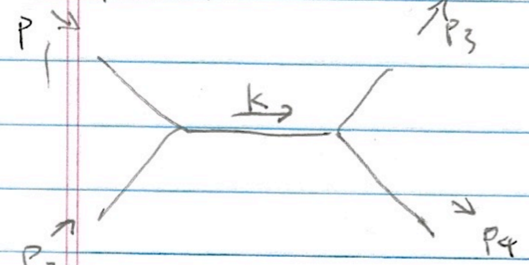
$$\Rightarrow \mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2$$

$$= \left[- \frac{\lambda^2 \, p_2^{\mu} (p_1+p_2)_{\mu} \, p_4^{\nu} (p_1+p_2)_{\nu}}{(p_1+p_2)^2 + i\varepsilon} - g^2 \frac{1}{(p_1+p_2)^2 + i\varepsilon} \right] \times \delta^4(\Sigma p)$$

Now we evaluate



In p-space:



$$iM_1 = i\lambda (p_2^\mu k_\mu) \frac{i}{k^2 + i\epsilon} i\lambda (p_4^\nu k_\nu) \delta(p_1 + p_2 - k) \delta(k - p_3 - p_4)$$

$$= \frac{(-i\lambda^2)\, p_2^\mu (p_1 + p_2)_\mu\, p_4^\nu (p_1 + p_2)_\nu \times \delta^4(\Sigma p)}{(p_1 + p_2)^2 + i\epsilon}$$

$$M_1 = \frac{(-\lambda^2)\, p_2^\mu (p_1 + p_2)_\mu\, p_4^\nu (p_1 + p_2)_\nu}{(p_1 + p_2)^2 + i\epsilon} \times \delta^4(\Sigma p)$$

Davidson Cheng

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